

POLYNOMIAL ROLLER COASTERS

Teaching Guidelines

Subject: Mathematics

Topics: Algebra, Polynomial Equations and Functions

Grades: 9 - 12

Knowledge and Skills:

- Can relate aspects of the graph of a polynomial function to the coefficients

Materials: (for each team)

- Graphing calculator or spreadsheet program

Procedure: This activity is best done by students working individually or in teams of two.

Students will need to already understand how to enter a polynomial function into a graphing calculator or spreadsheet program.

You may wish to simplify the activity by restricting the investigation to third or fourth-order polynomials from the beginning.

Answers:

1. Change the “ f ” coefficient from “ $-10,000$ ” to “ $-20,000$.” What is the effect on the roller coaster? Why? *The roller coaster drops to the right more steeply. “ f ” is the coefficient of the linear term, and when it has a negative value then that term imparts a negative slope to the graph. As f becomes more negative, that slope increases.*
2. What do you think would happen if you changed “ f ” to “ $-30,000$ ”? Try it and explain what happened. *As above.*
3. What happens if you change “ f ” to “ 0 ”? Explain. *The graph climbs more strongly to the right.*

4. Change all coefficients to “0” except for “ d ” (leave it at “20”). Describe the result. *The result is a standard cubic function graph, increasing from left to right.*
5. Now change “ d ” to “-20” (leaving all other coefficients equal to zero), and create the graph. Explain the results. *This inverts the graph, causing it to drop from left to right.*
6. Leave $d = -20$, and change “ f ” from “0” to “10000”. Try several other values of “ f ”, to see the effect. Describe the results. *This will cause “humps” to appear in the graph.*
7. Set “ a ” and “ b ” equal to zero, and see if you can find values for the other coefficients that produce a graph of this shape:

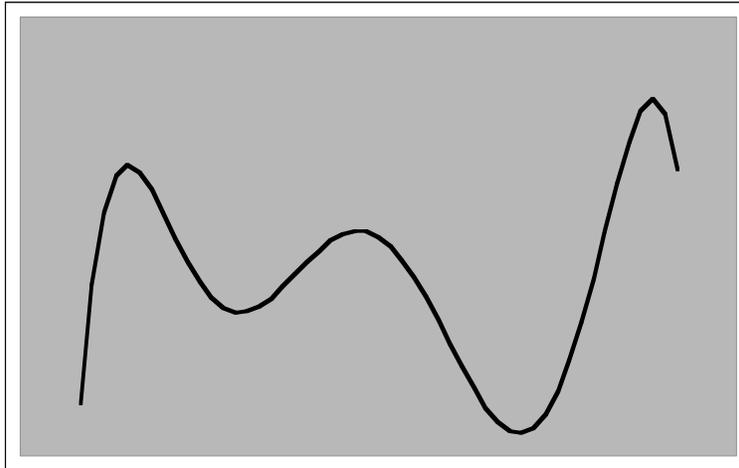
$$y = -14x^4 - 40x^3 + 10000x^2 + 10000x + 300000$$

Polynomial Roller Coasters

The shape of a roller coaster could be modeled by a polynomial function, such as this one:

$$y = ax^6 + bx^5 + cx^4 + dx^3 + ex^2 + fx + g.$$

Here is an example:



$$y = -.015x^6 + .01x^5 + 14x^4 + 20x^3 - 3000x^2 - 10000x + 300000.$$

(Domain: $-25 \leq x \leq 25$)

Use a graphing calculator or spreadsheet program to investigate the effects of the coefficients on the shape of the roller coaster, as follows:

1. Change the "f" coefficient from "-10,000" to "-20,000." What is the effect on the roller coaster? Why?

2. What do you think would happen if you changed "f" to "-30,000"? Try it and explain what happened.

3. What happens if you change " f " to "0"? Explain.

4. Change all coefficients to "0" except for " d " (leave it at "20"). Describe the result.

5. Now change " d " to "-20" (leaving all other coefficients equal to zero), and create the graph. Explain the results.

6. Leave $d = -20$, and change " f " from "0" to "10000." Try several other values of " f ", to see the effect. Describe the results.

7. Set " a " and " b " equal to zero, and see if you can find values for the other coefficients that produce a graph of this shape:

