SPACE IN SPACE

Teaching Guidelines

Subject: Mathematics

Topics: Problem Solving Strategies, Geometry

Grades: 7 - 12

Concepts

- Ratio (explicit)
- Non-linear variation (implicit)

Note: "Explicit" refers to concepts which are named and discussed in the course of the lesson; "implicit" refers to concepts which students experience in the course of the lesson but which are not named and discussed.

Knowledge and Skills:

- Can apply the problem-solving strategy "solve a simpler problem"
- Can apply the problem-solving strategy "draw a picture or diagram"
- Can apply the problem-solving strategy "predict and test" ("guess and check")
- Can determine the volume of a cube
- Can express ratios as percents

Procedure: This activity can be done by students individually or in teams of two.

Distribute the handout, and read through the first two paragraphs as a class. Discuss to ensure that students understand the information presented, and give students an opportunity to try to explain why the volume would change so much more than the diameter. (Accept all answers, but do not evaluate them.)

Review the problem solving strategy of "solve a simpler problem" and discuss to be sure students understand how it is being applied in this case. Then ask students to answer questions 1-3. (You may wish to review the formula for finding the volume of a rectangular prism and help students understand how that would apply to a cube.)

Circulate while students are working, to ensure they are computing correctly and that they understand how to create a ratio of the two numbers and convert that to a percent. [*Answers: volume of a 5.5 m cube is 166 cubic meters (rounded to nearest whole); volume of 5 meter cube is 125 cubic meters, ratio is 125/166 or about 75%, so the reduction in volume is 25%, or one-fourth.*]

Discuss results, and guide students to observe that while the percent of volume reduction was much greater than the percent of diameter reduction, it was not the 1/3 value expected. Then ask students to continue with the second page of the handout. Again, circulate and assist as needed while they are working on this. (Or you may choose to work through questions 4 to 7 as a class.)

[*Answers: #4: 5 meters, #5: 125 cubic meters; #6: 91 cubic meters (to nearest whole number); #6: reduction is about 28%*]

Discuss the answer to #7 as a class, and guide students to observe that although this is not yet 1/3 reduction, it is closer. Ask students to think about what would make the reduction even greater, and elicit or present the suggestion of making the walls even thicker. Then ask students to read question 8 and work to solve that posed problem. (You may or may not wish to suggest the problem solving strategy of "predict and test.") Again, circulate as students work.*

[*Answers: If the wall thickness is .75 meters, then the inside dimension of the 5.5 meter cube is 4.0 meters, with volume of 64 cubic meters, and the inside dimension of the 5 meter cube is 3.5 meters, with a volume of 42 cubic meters; the reduction in volume is then 33%.*]

As students reach the conclusion that the walls would have to be .75 meters thick, ask if that is a reasonable answer, and get out a meter stick so that students can easily envision how thick the walls would be. As a wrap-up, discuss possible reasons why the walls of the spacecraft might need to be that thick.

*Note: For more advanced students there is also an algebraic approach:

$$\frac{\text{Interior volume of 5 meter cube}}{\text{Interior volume of 5.5 meter cube}} = 67\% = .67$$

If *w* represents the thickness of the wall, then the interior dimension of the smaller cube is *5 – 2w*, and of the larger cube is *5.5 – 2w*. Since the volume of a cube is the cube of its dimension, you have:

$$\frac{(5-2w)^3}{(5.5-2w)^3} = .67$$

$$\frac{5-2w}{5.5-2w} = \sqrt[3]{.67} = .875$$

which can be solved straightforwardly for *w*.

Space in Space

In order to lower the original weight of the Orion vehicle, NASA engineers decided to reduce its size from an original diameter of 5.5 meters to a new diameter of 5 meters.

The diameter was reduced by a little under 10%, yet the habitable <u>volume</u> of the vehicle was reduced by about one third—around 33%. Why would that be?

To understand this, try the problem-solving strategy of "solve a simpler problem." In this case, instead of thinking about the cone-shaped Orion crew vehicle, think about what would happen to a simple cube:

1) Find the volume of a cube that has a length, width and height of 5.5 m.

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2) Find the volume of a cube that has a length, width and height of 5 m.

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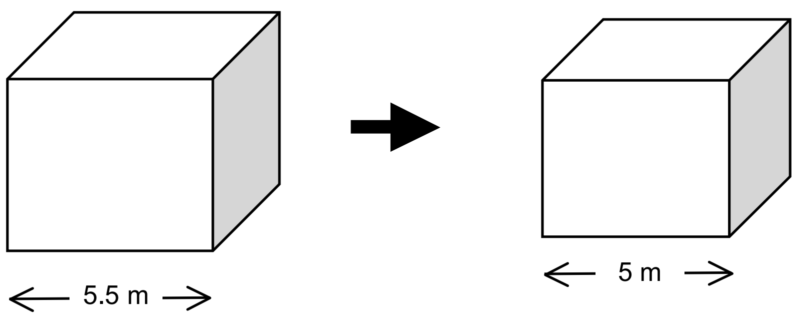

3) Compare the numbers (create a ratio, then convert to percent). What do you notice?

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Although this partly explains the reduction of volume, it's still not 1/3. But remember, it's the <u>habitable</u> volume that is reduced by 33%--that is, the volume <u>inside</u> the walls. Try this:

4) **Assume the walls of the cube are .25 m thick. If the outside dimension of the cube is 5.5 meters, what is the inside dimension? (Hint: Use the problem solving strategy of "draw a picture or diagram.")**

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5) **What is the inside volume?**

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6) **If the walls are .25 m thick, and the outside dimension is 5 meters, what is the inside dimension? What is the inside volume?**

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7) **Compare the answers to questions 4 and 6.**

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8) **Did that explain the 1/3 loss of volume? Not quite? Then solve this problem: <u>How thick</u> would the walls have to be to result in a loss of 1/3 habitable volume when the outside dimension of the cube is reduced from 5.5 m to 5 m? (What problem solving strategy can you use?)**

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9) **Is your answer reasonable? Explain.**

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